Technical Note

# DETERMINING THE CORRECT UNITS

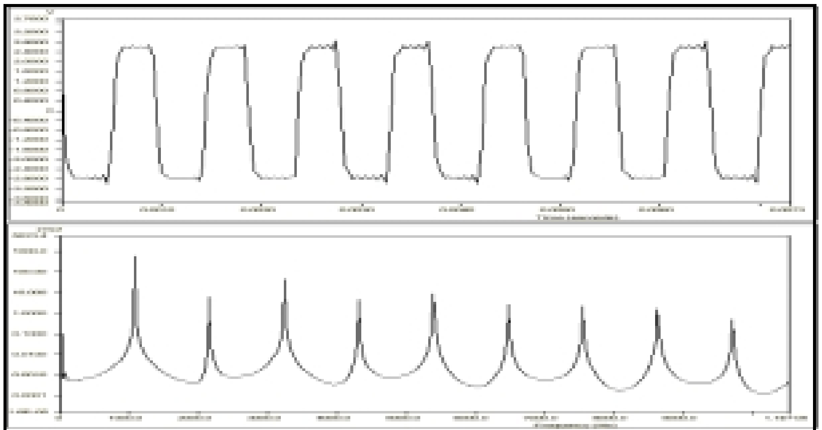

*Figure 1. Time capture and frequency spectrum for a stationary, deterministic signal. The frequency spectrum (bottom), is scaled as the root mean square of the signal.*

The Nicolet Prism software used with the Horizon and Compass Dynamic Signal Analyzer allows the user to choose the appropriate units to suit the measurement signal. The user can assign the correct units for deterministic, random or transient signals.

The amplitude scaling of the frequency spectra is affected by the record length (T) selected and the noise bandwidth (B). B = Df x k, where k is a scaling factor that depends on the choice of time window. The Prism software automatically compensates for these factors if the correct amplitude unit has been selected.

**Introduction to Signal Types**

Many different methods can be employed to apply FFT algorithms to a particular signal. The appropriate method largely depends on the signal type. There are two general categories of signal types: stationary and non-stationery. A stationary signal is independent of the particular sample record used to analyze it. While two stationary signals may look somewhat different, the results are equally valid for both. Deterministic Signals and Random Signals are the two types of stationary signals discussed in this note.

A non-stationary signal differs from a stationary signal in that each record of a non-stationary signal will contain unique information when analyzed. A Transient Signal is a prime example of a non-stationary signal. Non-stationary continuous signals also exist; however, that is beyond the scope of this application note.

**Deterministic Signals (Stationary)**

Stationary Deterministic Signals are made up entirely of sine waves at discrete frequencies. When the discrete frequencies are harmonically related, the signal is a periodic deterministic signal. When the distinct frequencies are not harmonically related the signal is a quasi-periodic deterministic signal. The unit selection is the same for both periodic and quasi periodic signals.

The resolution of the frequency analysis is determined by the filter bandwidth selected. The bandwidth should be selected to enable the analyzer to distinguish between the two most closely spaced frequency components. This means that there should only be one sinusoid in each filter passband at any one time. In such a case, the power transmitted by the filter is independent of the bandwidth. Therefore, the averaged frequency spectrum of a deterministic signal should be scaled in terms of root mean square (RMS) ($EU_{pk}$) or mean square power (PWR) $(EU_{rms})^2$.

The application of a time window on any signal distorts the amplitude of the signal. The end result of this is that the spectral amplitude of the FFT is not the true amplitude of the signal. However, this distortion can be corrected since we know the compensation factor appropriate for each window.

The Nicolet Prism software automatically makes the necessary adjustment when ($EU_{pk}$) or $(EU_{rms})^2$ is selected as the display unit. This allows for accurate cursor measurement of the peak of the spectrum, an important point of interest for deterministic signals. Note that if $(EU_{rms})^2$scaling is selected the reported amplitude of a spectral peak will be the square of the RMS value of the sinusoid at that frequency. If $EU_{pk}$ scaling is selected the reported amplitude is the peak value of the sinusoid at that frequency.

**Random Signals (Stationary)**

Any signal that varies in such a way as not to repeat itself is considered to be a random signal. A random signal has no time dependence; however, since its average properties do not change with time, it is considered a stationary signal. A FFT performed on such a signal is continuously distributed with frequency. Consequently, there is a continuous frequency distribution within the filter passband. As a result, the power transmitted by the filter is dependent on the filter bandwidth (B = Df x k).

While analyzing a stationary random signal, it may be necessary to remove the influence of the filter bandwidth. This can be done by dividing the transmitted power by the filter bandwidth. This normalizes the result to a mean square spectral density, often called the power spectral density (PSD). The PSD is a measure of the power per unit bandwidth. PSD can also be understood as a measure of the intensity in the frequency domain.

The PSD is usually computed from the FFT spectrum of a signal. Mathematically, a PSD can be described as the mean-square

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response of an ideal narrowband filter to x(t), divided by the bandwidth Df of the filter in the limit as Df (r) 0 at frequency f (Hz):

$$PSD(f) = \underset{\Delta f -> 0}{Lim} \frac{X_{\Delta f}^{2}}{\Delta f}$$

The amplitude distortion of signals during windowing was previously mentioned in this note. Another effect of windowing is the smearing of the spectra, where energy within the band of a single spectral line is smeared across several adjacent spectral lines. Hence, if the power of the individual spectra's were summed it would be an incorrect value. For random signals this is a significant alteration and point of concern.

The Nicolet Prism software automatically adjusts for this effect if any of the density functions ($(EU)^2$/Hz) are selected as the units for display. This ensures that the sum of the power is equal to the true power in the signal.

**Transient Signals (Non-Stationary)**

A transient is a signal which starts and finishes at zero. Transient signals contain a finite amount of energy and, therefore cannot be characterized in terms of power. This is because power is dependent on the record length: the longer the time capture the lower the average power. Like random signals, transient signals also typically have a spectrum continuously distributed with frequency. As a result, the transmitted power must be normalized to the filter bandwidth and re-scaled according to the record length. The result of this is the energy spectral density (ESD). The ESD is calculated by multiplying the PSD by the acquisition frame period, T. The discussion on choosing the correct density function for Random Signals applies to Transient Signals as well.

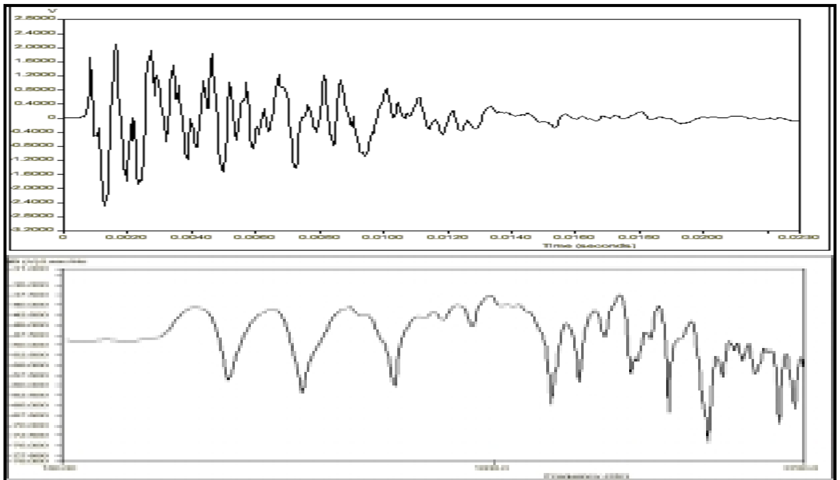

*Figure 3: Time capture and a frequency spectrum for a transient signal. The frequency spectrum is scaled as the Energy Spectral Density (ESD).*

**Conclusion**

All spectral methods are equally valid mathematically. Extracting accurate information from the analysis requires some knowledge of either the type of time-domain signal or a specific required result. Analysis of a signal using a Nicolet analyzer results in a frequency spectrum which is scaled correctly regardless of the frequency resolution and measurement time of the analysis.

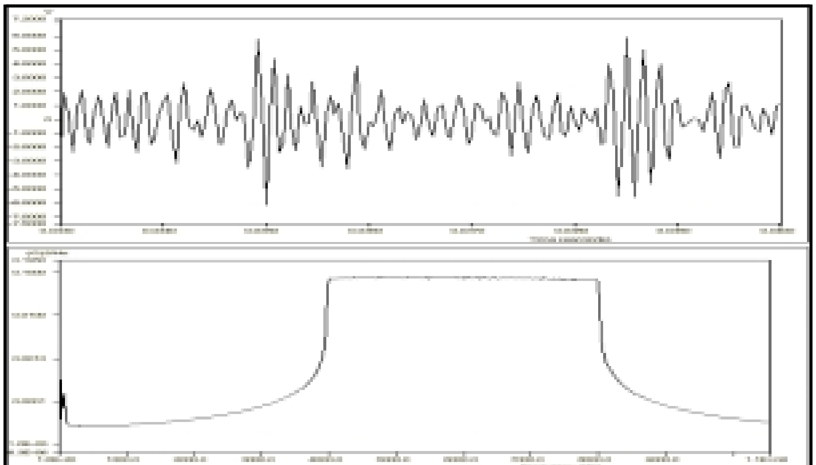

*Figure 2. Time capture and frequency spectrum for a stationary, random signal. The frequency spectrum is scaled as the power spectrum density (PSD).*